



ZoloBOSS™ Tomography Algorithm – An Overview

Introduction

The ZoloBOSS™ generates tomography based on a grid of path average measurements in order to depict the spatial distribution of temperature or a particular chemical species. In general, this rough tomography is generated in much the same way as the more complex computer aided tomography (CAT) scans used in medical imagery. This note provides an overview of the steps used to generate the ZoloBOSS™ tomography image without discussing the detailed algorithms used which are widely documented.¹ First, path intersection points are calculated based on the ZoloBOSS™ path average measurements. A low resolution image is generated from intersection points and finally, a smoothed, high resolution, image is created using methodologies similar to those used in CAT scans.

Path Intersection Points

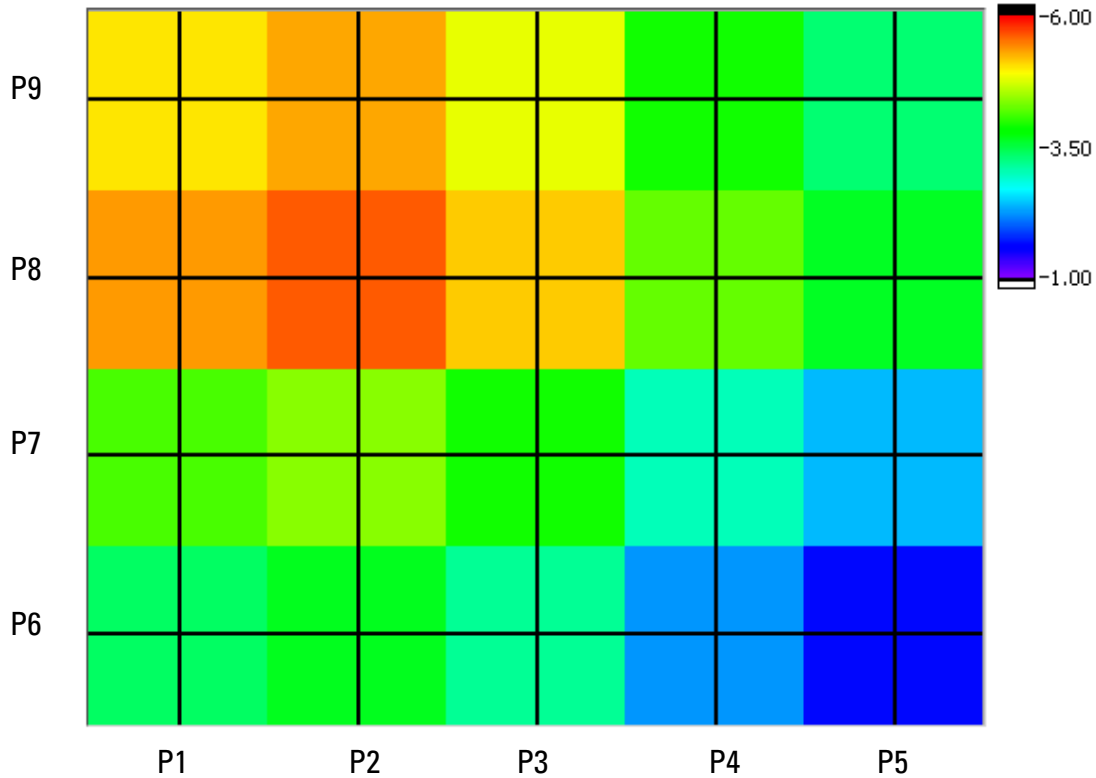
The simplest case is for equally spaced, rectilinear path averages. As an example the path average readings for a 5 x 4 path grid as shown in the table below.

N-S Paths	E-W Paths
P1=4.456	P6=2.785
P2=4.735	P7=3.547
P3=4.238	P8=4.788
P4=3.322	P9=4.457
P5=2.721	

The first step in going from one-dimensional path averages to a two-dimensional image is to apply an algorithm which generates values at the path intersection points. The grid intersection points are determined such that the average along a given path are as close as possible to the original measured path average value. The algorithm is designed to

¹ Chung, K. B., Gouldin, F. C. and Wolga, G. J., "Experimental reconstruction of the spatial density distribution of nonreacting flow with a small number of absorption measurements" Applied Optics Vol. 34, No. 24, pp 5492–5500, (1995)

minimize differences between two-dimensional grid point averages and all of the path averages. It is the root mean square (RMS) of the differences which is minimized.



The result of the first step in generating the Zolo**BOSS™** tomography leads to a picture as shown above. The inputs to the algorithm include nine path averages, paths 1 to 5 (P1 to P5) in the north-south direction intersecting with paths 6 to 9 (P6 to P9) in the east-west direction. The result is an image that includes values from the 20 path intersections. In this case, the paths are evenly spaced so each intersection point corresponds to an equal size grid box. The algorithm minimizes the difference between values at the grid intersections and the original path average values.

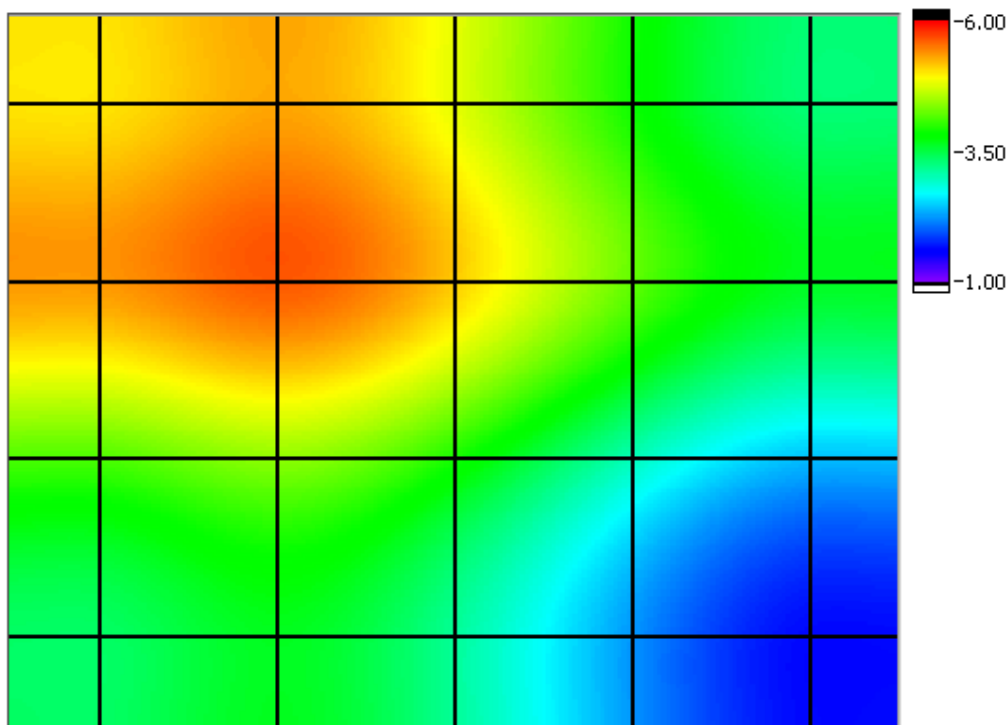
The low resolution “pixilated” image shown is one way in which the data could be displayed for the end user. However, the Zolo**BOSS™** software includes a final smoothing step to produce a continuous image while preserving the qualities of the low-resolution distribution.

Smoothed Image

Starting with the values at path intersection points, the next step in the Zolo**BOSS™** tomography algorithm is to assign values to the boiler walls using the value at the nearest grid point. This allows the known distribution to fill out the measurement plane beyond the outermost paths and assumes the distribution from the outermost paths to the boundary of the measurement plane is relatively flat.

In the absence of adding “boundary values” the smoothed image would only extend to the outermost paths and the resulting image would poorly reflect the low-resolution image as shown in the previous section. Appending the boundary values fills out the image so the appearance of the smoothed image is more consistent with the actual size of the measurement plane. Using this approach, the smoothed image values are nearly constant perpendicular to the boundary between the boundary and the nearest parallel path.

Continuing with the example shown in the previous section, the nearest-neighbor approach is used to assign boundary-path intersection points and then natural bi-cubic spline interpolation is used to fill in values between grid points. This leads to the following smoothed image.



The smoothed image accurately reflects the results of the low-resolution image shown in the previous section. The properties of the natural cubic spline are such that the values at the path intersection points are equal in both the low and high resolution images.

The nearest neighbor approach to assigning boundary values only effects those values between an outside path and the boundary, or boiler walls – where there are no path-path intersections points. In the example shown here, any point within the 12 internal cells is a valid estimate based on the Zolo **BOSS™** path average values. The value at a point within the 18 cells along the perimeter is equal to the value at the nearest point within the 12 internal cells.

Natural Cubic Spline

From Wikipedia:

A common spline is the **natural cubic spline** of degree 3 with continuity C^2 . The word natural means that the second derivatives of the spline polynomials are set equal to zero at the endpoints of the interval of interpolation.

The C^2 continuity refers to the fact that the first and second derivatives of the natural cubic spline are continuous everywhere, hence, the resultant interpolation appears very smooth to the human eye.

In essence, the Zolo **BOSS™** tomography algorithm assumes that the species distribution includes 2nd derivatives that are equal to zero at the path intersection points and the species distribution has continuous 1st and 2nd derivatives throughout the measurement plane. In general, cubic spline interpolation requires specifying additional conditions on the derivatives at the endpoints of the interval of interpolation. Lacking any additional information, the “natural” cubic spline is the least biased among the cubic spline interpolants.

An alternative approach would be to use a bi-linear interpolant. This would give rise to an image that would be similar to the natural cubic spline qualitatively, however, the resulting image would not be smooth – first derivative would be discontinuous at the path intersection points.

Summary

The Zolo **BOSS™** tomography algorithm requires the input of path average values. In general, the paths can be arbitrarily positioned, although in this note only equally spaced rectilinear paths are considered. The first step in the algorithm is to generate values at path intersection points which reconstruct the path averages. The next step is to assign nearest-neighbor values to boundary-path intersection points. Finally, to generate a smoothed image a natural bi-cubic spline interpolation is applied which assumes the 2nd derivatives at the path intersection points are equal to zero. This assumes the distribution of values is continuous (in the 1st and 2nd derivatives) throughout the measurement plane. The resulting smoothed image provides values at points between paths.